

# Affirmative answer to the Question of Leroy and Matczuk on injectivity of endomorphisms of semiprime left Noetherian rings with large images

V. V. Bavula

*School of Mathematical and Physical Sciences, University of Sheffield, Sheffield, UK*

## Abstract

The class of semiprime left Goldie rings is a huge class of rings that contains many large subclasses of rings – semiprime left Noetherian rings, semiprime rings with Krull dimension, rings of differential operators on affine algebraic varieties and universal enveloping algebras of finite dimensional Lie algebras to name a few. In the paper, ‘Ring endomorphisms with large images,’ *Glasg. Math. J.* **55** (2013), no. 2, 381–390, A. Leroy and J. Matczuk posed the following question:

*If a ring endomorphism of a semiprime left Noetherian ring has a large image, must it be injective?*

The aim of the paper is to give an affirmative answer to the Question of Leroy and Matczuk and to prove the following more general results.

**Theorem. (Dichotomy)** *Each endomorphism of a semiprime left Goldie ring with large image is either a monomorphism or otherwise its kernel contains a regular element of the ring ( $\Leftrightarrow$  its kernel is an essential left ideal of the ring). In general, both cases are non-empty.*

**Theorem.** *Every endomorphism with large image of a semiprime ring with Krull dimension is a monomorphism.*

**Theorem. (Affirmative answer to the Question of Leroy and Matczuk)** *Every endomorphism with large image of a semiprime left Noetherian ring is a monomorphism.*

## Keywords

Semiprime ring, semiprime left Goldie ring, left Noetherian ring, ring with Krull dimension, endomorphism with large image, monomorphism.