On Gröbnerian distributions of the escaliers of finite random points

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a joint work with Teo Mora

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Abstract

In the paper [3], the authors say that

> Often, randomly generated points will be in generic position, so it is easy to obtain these points for constructing examples.

But what are exactly *random* points? How the choices we make can affect the structures of the corresponding algebras?

To make some investigations on that, suppose to consider $(\mathbb{F}_3)^2$, which contains nine points in two coordinates. To each subset $\mathbf{X} \subseteq (\mathbb{F}_3)^2$ we can associate a vanishing ideal $I(\mathbf{X})$, with some reduced Gröbner basis $G(I(\mathbf{X}))$ and the related escalier $N(I(\mathbf{X}))$, which is a basis of the quotient algebra $\mathbb{F}_3[x, y]/I(\mathbf{X})$.

Take $|\mathbf{X}| = 3$. The different choices of size-three subsets $\mathbf{X} \subseteq (\mathbb{F}_3)^2$ gives rise to three different escaliers. We will study them by means of two of the main degröbnerization tools, that is, Cerlienco-Mureddu correspondence [1] (which is intrinsically lexicographic) and Möller algorithm [2] (which, instead, is not related to a specific ordering, so giving more freedom).

In this talk we will examine this situation in detail, showing probabilities of the occurrences for the different escaliers and some considerations on that.

Keywords

Cerlienco-Mureddu correspondence, Möller algorithm, quotient algebra.

References

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