A CHARACTERIZATION OF FINITE COMMUTATIVE FROBENIUS RINGS AND APPLICATIONS TO ALGEBRAIC CODING THEORY

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ABSTRACT. The fundamental alphabets for codes over rings are finite Frobenius rings, largely because their generating character produces MacWilliams relations. We prove that the following are equivalent statements for a finite commutative ring R: (1) R is Frobenius; (2) $|\mathfrak{a}||\mathfrak{a}^{\perp}| = |R|$ for all ideals \mathfrak{a} in R; and (3) $(\mathfrak{a}^{\perp})^{\perp} = \mathfrak{a}$ for all ideals \mathfrak{a} in R and give an algorithmic way of producing the generating character.

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