Monoids of relatively big projective modules and applications to direct sum decomposition of modules.

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Abstract

The study of finitely generated projective modules over a ring is a classical topic in ring and module theory. A set of representatives of their isomorphism classes can be given a monoid structure with the adition induced by the direct sum of module. The study of which kind of monoids can appear as the monoid of isomorphism classes of finitely generated projective modules over a ring is also a classical topic of study.

Recently, the attention has been turned also to the monoids of countably generated projective modules. In this talk we will present a particular kind of such monoids, the one of countably generated projective modules that are relatively big with respect to a trace ideal. This type of projective modules was introduced by P. Příhoda [4], as an extension of Bass' uniformly big projectives.

There is a surprising number of interesting examples of rings whose countably generated projective modules are always relatively big. This includes all right noetherian rings satisfying a polynomial identity (in particula, finitely generated algeb ras over a commutative noetherina ring), semilocal noetherian rings and some classes of algebras over hlocal domains.

These classes of rings have an interesting and relatively well established direct sum decomposition theory for finitely generated (presented) modules. The monoid of relatively big projectives gives a tool to extend this theory to infinite direct sums.

The content of the talk is based in our recent papers [3], [1] and [2].

Keywords

Rings, projective modules, relatively big projective modules, trace ideal, direct sum decomposition of modules.

References

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