Hopf braces, Hopf trusses and Hopf heaps

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Abstract

Here C denotes a coalgebra with counit (C, Δ, ε). A left Hopf truss consists of

(*) a coalgebra C with two algebra structures such that one makes C into a Hopf algebra $(C, \cdot, 1, \Delta, \varepsilon, S)$ and the other into a nonunital bialgebra $(C, \circ, \Delta, \varepsilon)$,

(**) connected by a coalgebra endomorphism $\sigma\colon C \to C$ such that $x \circ (y \cdot z) = \sum (x_1 \circ y) \cdot S\sigma(x_2) \cdot (x_3 \circ z)$ holds for all $x, y, z \in C$.

An equivalent formulation of the statement (**) is

(***) Define a ternary operation $[-,-,-]\colon C \otimes C^{\mathrm{cop}} \otimes C \to C$ by $[x, y, z] = x \cdot S(y) \cdot z$ for all $x, y, z \in C$. Then $x \circ [y, z, t] = \sum [x_1 \circ y, x_2 \circ z, x_3 \circ t]$ holds for all $x, y, z, t \in C$.

A ternary operation defined in the statement (***) makes C into a Hopf heap $(C, \Delta, \varepsilon, [-,-,-])$. A Hopf heap consists of a coalgebra C with a coalgebra homomorphism $\chi\colon C \otimes C^{\mathrm{cop}} \otimes C \to C$, $x \otimes y \otimes z \mapsto [x, y, z]$ such that $[[x, y, z], t, u] = [x, y, [z, t, u]]$ and $\sum [x_1, x_2, y] = \sum [y, x_1, x_2] = \varepsilon(x) y$ hold for all $x, y, z, t, u \in C$.

This talk is intended as a discussion of Hopf trusses and Hopf heaps.

Keywords

Coalgebras, Hopf algebras, Braces, Trusses.

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