Representation Graphs of Quadratic Forms

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a joint work with M. C. Zimmermann

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Abstract

Let q be a quadratic form on an A-module V and $a \in A$. We consider the Cayley graph on V with generating set $V_a = \{x \in A \mid q(x) = a\}$, i.e. the graph with vertices V in which $x, y \in V$ are adjacent if and only if $q(x-y) = a$. This graph is called the *representation graph of q with respect to a*.

A first example of such graphs is the unit distance graph on the Euclidean plane. This graph is subject to research for over 70 years and of particular prominence is the Hadwiger-Nelson problem: determine its *chromatic number*, i.e. the minimal number k which is needed to assign one of k colors to each vertex such that adjacent vertices are not of the same color. Further, in case of a finite field, representation graphs have been shown to be Ramanujan graphs in many cases ([3]) This shows that these graphs exhibit properties which makes them interesting for applications like coding theory.

While most results about representation graphs in the literature were obtained using combinatorial techniques, we will show how algebraic methods enable us to prove combinatorial results. In particular, we will show how decompositions of quadratic forms and the group of self-isometries can be used to determine graph invariants related to connectedness like *clique number*, *diameter* and *girth*.

Keywords

Graph, Quadratic Form, Clique, Diameter

References

[1] N. Lorenz, M. Zimmermann, Cliques in Representation Graphs of Quadratic Forms, arXiv preprint https://arxiv.org/abs/2306.07108 (2023).

[2] N. Lorenz, M. Zimmermann, Diameter and Girth of Representation Graphs of Quadratic Forms, arXiv preprint: https://arxiv.org/abs/2503.01721 (2025).

[3] A. Medrano, P. Myers, H. Stark, A. Terras, Finite analogues of euclidean space, Journal of Computational and Applied Mathematics 68 (1-2) (1996) 221–238.