Gröbnerian and Gröbner Free Techniques on Non-associative Algebra

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a joint work with M. Ceria

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Abstract

In this talk, we investigate to what extent Gröbnerian technologies and the Gröbner-free approach can allow us to describe the structure of non-associative magmas.

We consider a $\mathbb{F}-$vector space $V, \dim_{\mathbb{F}}(V) = n$, label and enumerate as $\mathbf{S} = \{x_1, \ldots, x_n\}$ an independent basis of it, and consider in V a non-associative binary operation $\circ : V \times V \mapsto V$ and the magma S generated by it.

In this setting, Gröbner bases have been studied by de Graaf—Wisliceny[**?**] and Gerritzen[**?**, **?**] who defined as Buchberger reduction the natural reduction and proved that any interreduced set is a reduced Gröbner basis G of the ideal $\mathbb{I}(G)$ it generates, so that neither S-pair completions nor criteria are needed.

In order to describe such ideal $\mathbb{I}(G)$ we make use of the alternative description proposed by Buchberger and Möller in terms of functionals[**?**]; this requires to define, for each $P := (a_1, \ldots, a_n) \in \mathbb{F}^n$, the functional $L_P : \mathbb{F}\{S'\} \to \mathbb{F}$, to give an adaptation of Möller Algorithm and state a proper (and trivial) Cerlienco–Mureddu Correspondence, which allows us to prove that

$$\mathbb{I}(G) = \{f \in \mathbb{F}\{S'\} : L_P(f) = 0, P \in \mathbb{F}^n\}.$$

This allows us to answer the challenging query of Pistone, Riccomagno and Rogantin: they consider the design ideal

$$\mathbb{I}(\mathcal{F}) := \{f \in \mathbf{k}[x_1, x_2] : f(a, b) = 0, (a, b) \in \mathcal{F}, \mathcal{F} := \{(0, 0), (1, -1), (-1, 1), (0, 1), (1, 0)\}$$

and the model $1, x_1, x_1^2, x_2, x_2^2$ which is the most symmetric of the models in the statistical fan but correctly remark that such model is not a monomial basis related to a Gröbner basis because *to distroy symmetry is a feature of Gröbner basis computation*. It was sufficient to remove the irrelevant restriction that the required monomial basis be related to some termording to allow us to produce a symmetric model for $\mathbb{I}(\mathcal{F})$.

Keywords

Non-associative magmas, Möller Algorithm, Design ideals

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