Invariant Subspaces and the Characterization Problem of Affine n-spaces

Z. Nazemian

University of Graz

a joint work with

H. Huang[1], Y. Wang[2], & J. J. Zhang[3]

[1] *Shanghai University, Shanghai, China*
[2] *Shanghai University of Finance and Economics, China*
[3] *University of Washington, Seattle, USA*

Abstract

In a Bourbaki seminar [1], Kraft identified the Characterization Problem for polynomial rings as one of the eight central challenges in affine algebraic geometry, alongside the Jacobian Conjecture, the Automorphism Problem, and the Zariski Cancellation Problem (ZCP).

We prove the following result (see [2]):

Let K be an algebraically closed field of characteristic zero, and let A be a (not necessarily commutative) algebra. Then $A \cong K[z_1, \cdots, z_m]$ for some $m \geq 2$ if and only if the following two conditions hold:

1. A is finitely generated and connected graded;
2. A has no nontrivial invariant subspaces.

Condition (1) is essential: for example, the Weyl algebras satisfy condition (2), but not condition (1), and are not isomorphic to polynomial rings. It remains an open question whether condition (1) can be replaced with a weaker assumption.

Keywords

Invariant subspaces, Polynomial rings

References

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