On equivalence of skew polycyclic codes over finite fields

Hassan Ou-azzou

Institute of Computer Science, University of St. Gallen, St. Gallen, Switzerland

a joint work with

Anna-Lena Horlemann¹,

¹ Institute of Computer Science, University of St.Gallen, St. Gallen, Switzerland

Abstract

We study the equivalence of families of skew polycyclic codes over a finite field \mathbb{F}_q , associated with skew polynomials $f(x) = x^n - a_{n-1}x^{n-1} - a_{n-1}x^{n-1}$ $\cdots - a_1 x - a_0 \in \mathbb{F}_q[x; \sigma]$, where σ is an automorphism of \mathbb{F}_q . We begin with the case of *skew trinomial codes* (or ℓ -trinomial codes), associated with polynomials of the form $f(x) = x^n - a_\ell x^\ell - a_0$, for some $0 < \ell < n$, and then extend our results to general skew polycyclic codes. We introduce an equivalence relation, called (n, σ) -equivalence, which generalizes the known notion of *n*-equivalence for (skew) constacyclic codes. We compute the number of (n, σ) -equivalence classes and provide conditions under which two families of skew polycyclic (or skew trinomial) codes are equivalent. In the special case where $\sigma = id$, we recover the equivalence of polycyclic codes, which generalizes known results on the equivalence of constacyclic codes. In this case, we show that when $gcd(n, n - \ell) = 1$, any ℓ -trinomial code family is equivalent to one associated with the canonical polynomial $x^n - x^{\ell} - 1$. We also study the case of p^{ℓ} -trinomial codes of length $p^{\ell+r}$, where p is the characteristic of \mathbb{F}_q and r is an integer, and we illustrate our results with several concrete examples.

Keywords

Skew polycyclic codes, Skew trinomial codes, skew constacyclic codes, Equivalent codes, Irreducible polynomials, skew polynomials.

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