Tilting Equivalences

from module categories to derived categories

Alberto Tonolo

Dipartimento di Matematica Pura ed Applicata
Università di Padova, Italy

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Outline

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Equivalence of Categories

A pair of covariant functors (F, G) between two additive categories $\mathcal{C}$ and $\mathcal{D}$ defines an equivalence

$$\mathcal{C} : \underset{G}{\overset{F}{\rightleftarrows}} : \mathcal{D}$$

if $\quad G \circ F \cong id_{\mathcal{C}}$ and $F \circ G \cong id_{\mathcal{D}}$,

i.e. for each C in $\mathcal{C}$ and D in $\mathcal{D}$ there exist natural isomorphisms

$$C \cong G(F(C)) \quad \text{and} \quad D \cong F(G(D)).$$

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Equivalence of Module Categories

Morita, 1958

Let R and S be two associative rings and

$$\text{Mod-}R \underset{F}{\overset{G}{\rightleftarrows}} \text{Mod-}S$$

an equivalence of categories. Then the right R-module $V_R := G(S)$ is a progenerator, i.e.

(1) V_R is a direct summand of R^{n_0}, $n_0 \in \mathbb{N}$;

(3) R is a direct summand of $V_R^{m_0}$, $m_0 \in \mathbb{N}$.

Moreover

$\bullet S = \mathrm{End}(V_R)$ $\bullet F \sim \mathrm{Hom}_R(V, -)$ $\bullet G \sim - \otimes_S V$.

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Equivalence of Module Categories

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Conversely any progenerator V_R induces an equivalence

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From progenerators to classical tilting modules

In other redundant words, a right R module V_R is a progenerator if

(1) there exists an exact sequence

$$0 \to P_0 \to V \to 0$$

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From progenerators to classical tilting modules

In other redundant words, a right R module V_R is a progenerator=classical 0-tilting if

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From progenerators to tilting modules

A right R module V_R is classical 1-tilting if

(1) there exists an exact sequence

$$0 \to P_1 \to P_0 \to V \to 0$$

with $P_i \leq^{\oplus} R^{n_i}$, where $n_i \in \mathbb{N}$, $i = 0, 1$;

(2) $\mathrm{Ext}^i_R(V, V) = 0$ for each $i \geq 0$;

(3) there exists an exact sequence

$$0 \to R \to V_0 \to V_1 \to 0$$

with $V_i \leq^{\oplus} V^{m_i}$, where the $m_i \in \mathbb{N}$, $i = 0, 1$.

From progenerators to classical tilting modules

A right R module V_R is classical 2-tilting if

(1) there exists an exact sequence

$$0 \to P_2 \to P_1 \to P_0 \to V \to 0$$

with $P_i \leq^{\oplus} R^{n_i}$, where the $n_i \in \mathbb{N}$, $i = 0, 1, 2$;

(2) $\mathrm{Ext}^i_R(V, V) = 0$ for each $i \geq 0$;

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$$0 \to R \to V_0 \to V_1 \to V_2 \to 0$$

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From progenerators to classical tilting modules

A right R module V_R is classical n-tilting if

(1) there exists an exact sequence

$$0 \to P_n \to \dots \to P_1 \to P_0 \to V \to 0$$

with $P_i \leq^{\oplus} R^{n_i}$, where the $n_i \in \mathbb{N}$, $i = 0, 1, \dots, n$;

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Tilting equivalences

Let V_R be a classical n-tilting module and $S := \mathrm{End}(V)$. We want to study the equivalences induced by the functors

- $\mathrm{Hom}_R(V, -) : \text{Mod-}R \to \text{Mod-}S$
- $- \otimes_S V : \text{Mod-}S \to \text{Mod-}R$

and their derived functors.

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Notation

In the sequel V_R is a classical n-tilting module and $S := \mathrm{End}(V_R)$; moreover

- $E^i := \mathrm{Ext}^i_R(V, -)$;
- $T_i := \mathrm{Tor}^i_S(-, V)$;
- $KE_V(i) := \bigcap_{j \neq i} \mathrm{Ker}\, \mathrm{Ext}^j_R(V, -)$;
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$n = 1$

Brenner and Butler 1981, Colby and Fuller 1990

- There are two equivalences

$$KE_V(i) \underset{T_i}{\overset{E^i}{\rightleftarrows}} KT_V(i), \quad i = 0, 1$$

Each right R-module M and each right S-module N are the central terms of short exact sequences

$$0 \to \underbrace{M_0}_{\in KE_V(0)} \to M \to \underbrace{M_1}_{\in KE_V(1)} \to 0 \text{ and}$$

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$n \geq 1$

Miyashita 1986

- There are $n+1$ equivalences

$$KE_V(i) \underset{T_i}{\overset{E^i}{\rightleftarrows}} KT_V(i), \quad i = 0, 1, ..., n$$

It is not anymore possible to approximate modules with objects of the subcategories involved in the equivalence theory.

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Miyashita 1986

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An example

$$R = 1 \xrightarrow{\phi} 2 \xrightarrow{\psi} 3 \text{ with } \psi \circ \phi = 0$$

Ind. Mod-R = $1,\ 2,\ 3,\ \begin{smallmatrix}3\\2\end{smallmatrix},\ \begin{smallmatrix}2\\1\end{smallmatrix}$, $V_R = \begin{smallmatrix}3\\2\end{smallmatrix} \oplus \begin{smallmatrix}2\\1\end{smallmatrix} \oplus 3$, $R \cong \mathrm{End}(V_R)$

$KE_V(0)$ $\begin{smallmatrix}3\\2\end{smallmatrix}$ $\begin{smallmatrix}2\\1\end{smallmatrix}$ 3

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$$\begin{matrix} & \begin{smallmatrix}3\\2\end{smallmatrix} \xleftarrow{\ E^0\ } \dashrightarrow_{T_0} \begin{smallmatrix}2\\1\end{smallmatrix} & \\ KE_V(0) & \begin{smallmatrix}2\\1\end{smallmatrix} \xleftarrow{\ E^0\ } \dashrightarrow_{T_0} 1 & KT_V(0) \\ & 3 \xleftarrow{\ E^0\ } \dashrightarrow_{T_0} \begin{smallmatrix}3\\2\end{smallmatrix} & \end{matrix}$$

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$$KE_V(2) \qquad 1 \xleftarrow[T_2]{E^2} \dashrightarrow 3 \qquad KT_V(2)$$

Why the simple 1 and not the simple 2?

If we consider the injective resolutions of 1 and 2 we get:

$$0 \to 1 \to \begin{matrix} 2 \\ 1 \end{matrix} \to \begin{matrix} 3 \\ 2 \end{matrix} \to 3 \to 0 \quad \text{and} \quad 0 \to 2 \to \begin{matrix} 3 \\ 2 \end{matrix} \to 3 \to 0.$$

Applying the functor $\mathrm{Hom}_R(V, -)$ to the injective resolutions we get

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The first is a projective resolution of the simple module 3, while the second is not a projective resolution of a module! It is a complex of projective modules with two non zero cohomologies.

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$\mathcal{D}(R)$ and $\mathcal{D}^b(R)$

The objects of the derived category $\mathcal{D}(R)$ are complexes of R-modules; the morphisms are obtained from complex homomorphisms in two steps:

1. identifying the homotopic homomorphisms,
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Extending the Hom functor

If V_R is a classical n-tilting module, it is possible to extend the functor $\mathrm{Hom}_R(V,-)$ and its derived functors $\mathrm{Ext}^i_R(V,-)$, $i \geq 0$, to a unique functor

$$\mathbb{R}\,\mathrm{Hom}(V,-) : \mathcal{D}(R) \to \mathcal{D}(S)$$

If M is a right R-module, $\mathbb{R}\,\mathrm{Hom}(V,M)$ is a complex of right S-modules and

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Analogously, the functor $-\otimes_S V$ and its derived functors $\mathrm{Tor}_i^S(-, V)$, $i \geq 0$, extend to a unique functor

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Happel 1987, Cline, Parshall and Scott 1986

A classical n-tilting module V_R induces a derived equivalence

$$\mathbb{R}\operatorname{Hom}_R(V,-) : \mathcal{D}^b(R) \rightleftarrows \mathcal{D}^b(S) : \mathbb{L}(- \otimes_S V)$$

From classical tilting to (good) tilting modules

Let us recall that a right R module V_R is classical n-tilting if

(1) there exists an exact sequence

$$0 \to P_n \to \ldots \to P_1 \to P_0 \to V \to 0$$

with $P_i \leq^{\oplus} R^{(\alpha_i)}$, where the α_i's are finite cardinals, $i = 0, 1, \ldots, n$;

(2) $\mathrm{Ext}^i_R(V, V) = 0$ for each $i \geq 0$;

(3) there exists an exact sequence

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Tilting modules

A right R module V_R is *n-tilting* if

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For any n-tilting module V_R there exists a good n-tilting module T_R *equivalent to* V_R, i.e. such that

$$KE_V(i) = KE_T(i), \text{ for each } i \geq 0$$

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Colpi and Trlifaj, 1995

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Bazzoni, Mantese and Tonolo, 2009

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