Plan:

A) Coding Theory - introduction
   1) Definitions, sphere packings & other bounds.
   2) Linear codes.
   3) Polynomial & cyclic codes.
   4) MacWilliams identity & theorem.

B) Finite extension and pseudo-linear maps.
   1) Definition of PLT.
   2) Polynomial maps & PLT.
   3) More results on polynomial maps.

C) Saw codes
   1) Utility of codes based on saw polynomials.
   2) $\mathbb{F}_q[x; \Theta]$ and factorizations.
   3) Codes based on $\mathbb{F}_q[x; \Theta]$.
   4) $(0, S)$ codes.

D) Codes over Frobenius rings
   1) Frobenius rings & algebras.
   2) Codes over commutative Frobenius rings.
   3) MacWilliams identity & extension theorem.
   4) Perspectives.
A) Coding Theory - Introduction

1) Definitions

\[
\begin{array}{c}
m \\ F^{m} \\ \downarrow \text{1-1} \quad e \\ m_e \\ \downarrow \text{error} \quad t \\ m_n \\ \downarrow F \quad F^{n} \\ \end{array}
\]

\[m > h \quad k \text{ rate} \quad k \text{ dimension} \quad n = \text{length}\]

**EX:** 1) Parity check \( n = h + 1 \)

\[F^{h} \rightarrow F^{h+1} \]

Add one bit to the message so that the sum of all bits is 0 (in \( F_{2} \)).

2) Repeating the words: \( n = 2h \)

Moreover, when you increase \( n \), you can detect more errors, but this costs!

(Some surprise: Shannon (1948), information theory)

**Def:** a) \( x, y \in F^{m}_{q} \)

\[d(x, y) = \#\{1 \leq i \leq m \mid x_i \neq y_i\}\]

b) \( E \subset F^{m}_{q} \)

A code

\[d(E) = \min \{d(x, y) \mid x, y \in E\}\]

\[t := \left\lfloor \frac{d-1}{2} \right\rfloor\]

Remark \( x, x' \in E \)

\[B(x, t) \cap B(x', t) = \emptyset\]

where \( B(x, t) = \{x \in F^{m}_{q} \mid d(x, c) \leq t\} \)

**EX:** \( E = \{c_1, c_2, c_3, c_4\} \)

Decoding 4 possibilities
Theorem: Let $C$ be a code with minimal distance $d$. Let $t = \left\lfloor \frac{d-1}{2} \right\rfloor$. Then:

a) $C$ can detect up to $d-1$ errors.
b) $C$ can correct up to $t$ errors.

Problem of coding theory: Let $C$ be a $(n, M, d)$ code where $|C| = M$.

1. $n$ must be small (costs and speed)
2. $M = |C|$ must be big (big variety of messages)
3. $d$ must be big (to detect & correct lots of errors)

Bounds: $C \subset \mathbb{F}^n_q$, $M = |C|$,

a) $M (1 + \binom{n}{1}(q-1) + \binom{n}{2}(q-2) + \cdots + \binom{n}{t}(q-1)^t) \leq q^m$ (Hamming bound)

b) $A_q(m, d)$ maximal number of code words for a code $C \subset \mathbb{F}^n_q$ such that $d = d(C)$ then $A_q(m, d) \leq q^{n-d+r}$

e) Linear codes.

def: A code $C \subset \mathbb{F}^n_q$ is linear if $C$ is a vector space.

$\dim C = \lambda$, $M = \left| C \right| = q^\lambda$, $x \in C$, $\omega(x) = \{ i \mid x_i \neq 0 \}$

Property: $C \subset \mathbb{F}^n_q$, $d(C) = \min \{ \omega(x) \mid x \in C \}$
Definitions: Let a linear \((m, k)\)-code \(C\).

a) A generic matrix is a \(k \times m\) matrix over \(\mathbb{F}_q\) such that the rows form a basis of \(C\).

b) Dual of \(C\) is \(C^\perp = \{ x \in \mathbb{F}_q^m \mid x \cdot y = 0 \ \forall y \in C \}\)
   where, for \(x = (x_1, \ldots, x_m), \ y = (y_1, \ldots, y_m), x \cdot y = \sum_{i=1}^{m} x_i y_i\).

c) A control matrix for \(C\) is a generic matrix for \(C^\perp\). Since \(\dim C + \dim C^\perp = n\)
   \[H \in M_{(n-k) \times n}(\mathbb{F}_q)\]
   \[HG^t = 0 \quad GH^t = 0\]
   \[E = \{ x \in \mathbb{F}_q^m \mid Hx = 0 \}\]
   \[C^\perp \perp = C\]

3) Cyclic codes

Definition: A linear code \(C \subseteq \mathbb{F}_q^m\) is cyclic if

\[(a_0, \ldots, a_{m-1}) \in C \Rightarrow (a_{m-1}, a_0, \ldots, a_{m-2}) \in C\]

We attached to \((a_0, \ldots, a_{m-1}) \in C\) the polynomial

\[\sum_{i=0}^{m-1} a_i x^i \in \mathbb{F}_q[x]/((x^m - 1)/(x - 1))\]

\(x = x + (x^{m-1})\)
Hence \((a_0, \ldots, a_{m-1}) \in \mathbb{F}_q \implies g(x) = \sum a_i x^i \in \mathbb{F}_q[x]/(x^m - 1)\)

\((a_{m-1}, \ldots, a_0) \in \mathbb{F}_q \implies g(x) x\)

In this way cyclicity corresponds to stability by left multiplication by \(x\).

\(\mathbb{F}_q^m\) cyclic codes \(\iff\) ideals of \(\mathbb{F}_q[x]/(x^m - 1)\)

\(\mathbb{F}_q^m\) cyclic codes \(\iff\) factors of \(x^m - 1\)

\(\mathbb{F}_q^m \triangleleft \mathbb{F}_q[x]\) \(\iff\) \(g(x) \in \mathbb{F}_q[x]\) \(g(x) \mid x^m - 1\)

\(\dim E = h \iff \deg g(x) = m - h\)

**Theorem**

\(E \subseteq \mathbb{F}_q^m\) a cyclic code with associated polynomial \(g(x) = g_0 + g_1 x + \cdots + g_{m-1} x^{m-1}\)

The generic matrix is given by

\[
G = \begin{pmatrix}
    g_0 & g_1 & \cdots & g_{m-1} \\
    0 & g_0 & g_1 & \cdots \\
    \vdots & \ddots & \ddots & \ddots \\
    0 & \cdots & 0 & g_0 \\
    \vdots & \cdots & \cdots & \cdots
\end{pmatrix} \in M_{2 \times m}(\mathbb{F}_q)
\]

the control matrix (also as parity check matrix)

\[
H = \begin{pmatrix}
h_0 & h_1 & \cdots & h_{m-1} & 0 & \cdots & 0 \\
0 & h_0 & h_1 & \cdots & h_{m-1} & 0 & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{m-1}
\end{pmatrix} \in M_{m-h \times m}(\mathbb{F}_q)
\]

where \(g(x) h(x) = x^m - 1\) \(\implies h(x) = h_0 + h_1 x + \cdots + h_{m-h} x^{m-h}\)

check polynomial.
Example  4  Golay code  $X^{23} - 1 = (X-1)g(x)g_1(x)$

where  $g(x) = X^{14} + X^9 + X^7 + X^6 + X^5 + X + 1$

$g_1(x) = X^{14} + X^{10} + X^6 + X^5 + X^4 + X + 1$

These two polynomials are reciprocal of each other ($g_1(x) = X^{14}g(x^{-1})$ and $g(x) = X^g_1(x^{-1})$) and hence generate equivalent codes -- these codes are cyclic (23, 12) - codes with generator $g(x) \in \mathbb{F}[x]_{23}$. The code word corresponding to $g(x)$ is of weight 7 and in fact it gives the minimal weight i.e., $d(\mathcal{C}_{23}) = 7$. It can correct up to 3 errors.

$\mathcal{C}_{23}$ is a perfect code (The sphere packing bound is attained).

Corollary  If $\mathcal{C} \subset \mathbb{F}^n$ is a cyclic code then $\mathcal{C}^\perp$ is cyclic as well. (with generating polynomial equal to the reciprocal of the annihilator of the generator of $\mathcal{C}$)
4) MacWilliams identity and theorem

Definition: \( \mathcal{E} \subset \mathbb{F}_q^m \) a linear code. The weight enumerator polynomial is \( W_{\mathcal{E}}(x, y) = \sum_{i=0}^{\mathbb{F}_q} \omega_i \cdot x^{i-q} y^i \) where \( \omega_i \) is the number of codewords in \( \mathcal{E} \) of weight \( i \).

Example: \( \mathcal{E} = \{000, 111\} \subset \mathbb{F}_2^3 \), \( W_{\mathcal{E}}(x, y) = x^3 + y^3 \)

\( \mathcal{E}^\perp = \{000, 011, 010, 110\} \), \( W_{\mathcal{E}^\perp}(x, y) = y^3 + 3x^2y^2 \)

Theorem (MacWilliams) 1963: \( \mathcal{E} \subset \mathbb{F}_q^m \) linear

\( W_{\mathcal{E}^\perp}(x, y) = \frac{1}{|\mathcal{E}|} W_{\mathcal{E}}(x + (q-1)y, x - y) \)

Example 1: \( \mathcal{E} = \{000, 111\} \), \( \mathcal{E}^\perp = \{000, 110, 101, 011\} \)

\( W_{\mathcal{E}}(x, y) = x^3 + y^3 \), \( W_{\mathcal{E}^\perp}(x, y) = x^3 + 3x^2y^2 \)

Example 2: \( \mathcal{E} \) the linear code of length 8 with generic matrix

\[ G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ \end{pmatrix} \]

\( \mathcal{E}^\perp \)

\( W_{\mathcal{E}} = x^2 + 14x^4y^2 + y^8 = W_{\mathcal{E}^\perp} \)

Definitions:

1. \( T: \mathbb{F}_q^m \rightarrow \mathbb{F}_q^m \) is called a monomial transformation if there exists \( \sigma \in S_m \) and \( \mu_1, \ldots, \mu_m \in \mathbb{F}_q^* \) such that

\[ T(x_1, \ldots, x_m) = (\sigma(x_1)\mu_1, \ldots, \sigma(x_m)\mu_m) \]

2. \( \mathcal{E}_1 \subset \mathcal{E}_2 \subset \mathbb{F}_q^m \) linear codes are monomially equivalent if there exists a mon. \( T \) s.t. \( T(\mathcal{E}_1) = \mathcal{E}_2 \)
A linear isomorphism \( f : E_1 \to E_2 \) between linear codes in \( \mathbb{F}_q^m \) is an isometry if it preserves Hamming weight:

\[
\text{wt}(f(x)) = \text{wt}(x)
\]

**MacWilliams Extension Theorem**: \( E_1, E_2 \) are two linear codes in \( \mathbb{F}_q^m \), \( f : E_1 \to E_2 \) a linear isomorphism. Then:

\( f \) is an isometry if \( f \) extends to a monomial transform of \( \mathbb{F}_q^m \)

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**B) One extensions and pseudo-linear maps**

1. Let \( A \) be a ring, \( \sigma \in \text{End } (A) \) a \( \sigma \)-derivation of \( A \) and \( R = A[t; \sigma, \delta] \). If \( V \) is a left \( R \)-module then \( V \) is a left \( A \)-module and

\[
\sigma(a)v = \sigma(a)v + \delta(a)v = t(\sigma v)
\]

This motivates: \( T : V \to V \) is a \((\sigma, \delta)-\text{PLT}\) if \( T \) is \( A \)-linear and

\[
T(av) = \sigma(a)T(v) + \delta(a)v
\]

There is a correspondence between

left \( R \)-modules \( \leftrightarrow \) \((\sigma, \delta)-\text{PLT}\) on left \( A \)-modules

**Definition**

\[
f(t) \in R = A[t; \sigma, \delta], \quad a \in A \quad f(a) \in A
\]

\[
defined \text{ by } f(t) - f(a) \in R(t - a)
\]
Propriétés

$f(t), g(t) \in \mathbb{R}$ a.e $A$ $T_{a}$ $S_{x}$ P.L.T

a) $f(T)g(T) = (fg)(T)$, where $(\sum_{x \in T}e_{x})(T) = \sum_{x}e_{x}T$

b) $f(a) = f(T_{a})(1)$, where $T_{a}(x) = e_{a}(x)a + S(x)$

c) $(fg)(a) = f(T_{a})(gca)$

d) If $g(a) \in U(A)$ $(fg)(a) = f(a^{g(a)})g(a)$

$\text{where } a^{e} = e_{c}(a)a^{-1} + S(e)a^{-1} x \in U(A)$

Applications to finite extensions over finite fields.

We can restrict to finite extensions $\mathbb{F}_{q}[x; \sigma]$

where $\sigma \in \text{Aut}(\mathbb{F}_{q})$.

Let $\Theta: \mathbb{F}_{q} \rightarrow \mathbb{F}_{q}$ be the Frobenius automorphism

$\Theta(a) = a^{q}$, $a \in \mathbb{F}_{q}$

Let $x$ be a commuting indeterminate over $\mathbb{F}_{q}$, we can extend $\Theta$ to $\mathbb{F}_{q}[x]$ & consider $\mathbb{F}_{q}[x][t; \Theta]$

$(\Theta(x) = x^{q})$. We note:

a) For $i > 1$, $x^{i} = \frac{x^{i-1}}{p-1} = p + p + \cdots + p + 1$

and put $[0] = 0$.

b) $\mathbb{F}_{q}[x^{e}] = \{ \sum_{x \in \sigma} x^{e} x^{i} \in \mathbb{F}_{q}[x] \}$

We can consider evaluation at $x$ & get the evaluation map: $\mathbb{F}_{q}[x][t; \Theta] \rightarrow \mathbb{F}_{q}[x]$ and hence
This is used in connection with \( T_\mathfrak{m} \) to get the factorizations results in the second part of the following Theorem (L.)

A) The polynomial \( G(t) = t^{(p-1)m+1} - t \) is the least common multiple of \( t - a_i, a_i \in \mathbb{F}_q \) in \( \mathbb{F}_q[t; \Theta] \). We have \( RG = GR \).

B) For \( a \in \mathbb{F}_q \), \( f(t) = \sum a_i t^{i} \in R = \mathbb{F}_q[t; \Theta] \)

1) \( f(b) = \sum a_i b^{i} \)

2) \( f(t)(x) = \sum a_i x^{i} = x \sum a_i x^{i} \)

3) \( \{ f^{i} | f \in R = \mathbb{F}_q[t; \Theta] \} = \mathbb{F}_q[x^{i}] \)

4) \( f(t) \in R = \mathbb{F}_q[t; \Theta], f(t) \in RH(t) \) if and only if \( f^{i} \in \mathbb{F}_q[x^{i}] \)

5) factorization of \( f(t) \in \mathbb{F}_q[t; \Theta] \) can be obtained from factorization in \( \mathbb{F}_q[x] \)

6) \( f(t) \) is irreducible in \( R \) iff \( f^{i} \) is \( \mathbb{F}_q[x^{i}] \) irreducible

There are many more factorizations in \( \mathbb{F}_q[t; \Theta] \)

then in \( \mathbb{F}_q[x] \). This is one of the reasons that these rings were used in coding theory (Bocher, Sch. Ulma,...)

We will show how the PLTs seem to be useful in the more general frame of \( R = A[t; s, \delta] \)
E) Codes with skew polynomials.

Ulmer, Boucher, Sole, ... used $\mathbb{F}_q[x; \sigma]$ to construct codes. The advantage is that there are a lot of distinct factorizations of a given polynomial.

Example: $x^4 + x^2 + 1 = (x^2 + x + 1)^2$ in $\mathbb{F}_q[x; \sigma]$

\[ (x^2 + x + 1)(x^2 + 1) = (x^2 + x + 1)(x^2 + 1)^2 = (x^2 + 1)^2 x + 1 = (x^2 + 2x + 1)^2 \]

$\sigma^2 + \sigma + 1 = 0$

Hence they discovered new (cyclic) codes with larger distances than the known codes of the same length.

Using P.L.T it is possible to go further:

Definition. $f \in R = A[t; \sigma, \delta]$ monic of degree $n$.

A $(\sigma, \delta)$-code $E \subseteq A^n$ is the set of $n$-tuples corresponding to the left $R$-module $R_f$ where $g/f$, $\deg g = n$. We get back cyclicity & generic matrix:

Theorem (Bouyjaneau, J.)

a) $v = (a_0, ..., a_{n-1}) \in E \Rightarrow f(v) \in E$

b) The $n$-tuples defined by $(f^k)(g_0, ..., g_n)$ for $0 \leq k \leq n-1$ generate the code $E$. 

\[ \sigma^2 + \sigma + 1 = 0 \]
This gives back the \( \mathcal{O} \)-codes defined over \( \mathbb{F}_r \):

\[(a_0, \ldots, a_{n-1}) \in \mathcal{O} \rightarrow (\mathcal{O}(a_0), \ldots, \mathcal{O}(a_{n-1})) \in \mathcal{E}\]

\[G = \begin{pmatrix}
    g_0 & g_1 & \cdots & g_{n-1} \\
    0 & \mathcal{O}(g_0) & \cdots & \mathcal{O}(g_{n-1}) \\
    0 & 0 & \mathcal{O}(g_0) & \cdots & \mathcal{O}(g_{n-1}) \\
    \vdots & \vdots & \ddots & \ddots & \vdots \\
    0 & 0 & \cdots & \mathcal{O}(g_0) & \cdots
\end{pmatrix}
\]

If we write \( \mathcal{O} = \mathcal{O}(h) \) in \( \mathcal{R} \), the code \( \mathcal{E} \) corresponds to \( \mathcal{R}/\mathcal{O} \).\( \mathcal{H} \) this can be used to get a control map, i.e., \( (a_0, \ldots, a_{n-1}) \in \mathcal{E} \leftrightarrow \sum_i T_f (a_i) = 0 \)

Another way of building codes with \( \mathcal{R} = \mathbb{A}[t; s, t] \) is to consider \( L \leq \mathbb{P} \) of polynomials \( t-a_0, \ldots, t-a_n \). The control map is easier to get since we just need to verify that \( a_i, \ldots, a_n \) are roots...
D) Looks over Frobenius rings.

1) Frobenius rings & algebras.

Theorem - Definition

A ring $R$ is a quasi-Frobenius ring if it satisfies the following equivalent conditions:

1. $R$ is right noetherian and right self-injective.
2. $R$ is left noetherian and right self-injective.
3. $R$ is right noetherian and satisfies the following double annihilator conditions:
   \[ \text{ann}_R(\text{ann}_R(I)) = I \text{ for any right ideal } I \subseteq R \]
   \[ \text{ann}_R(\text{ann}_R(J)) = J \text{ for any left ideal } J \subseteq R \]
4. $R$ is (two-sided) artinian and satisfies (3a) and (3b).
5. A right $R$-module is projective iff it is injective.

Let $J = \text{rad}(R)$ be the Jacobson radical of a ring $R$.

Then the following are equivalent:

(i) $R$ is Q.F. and $\text{soc}(R_R) \cong (R/J)_R$
(ii) $R$ is Q.F. and $\text{soc}(R) \cong R(J)$
(iii) $\text{soc}(R_R) \cong (R/J)_R$ and $\text{soc}(R) \cong (R/J)$
Examples

1) Any semisimple ring is a Frobenius ring.

2) \( \mathbb{Z}/m\mathbb{Z} \)

3) \( \mathbb{H}[f] \quad (f \neq 0) \)

4) If \( R \) is commutative, \( R \) is Frobenius if and only if \( R \cong \prod_{i} R_i \) where each \( R_i \) is a local Artinian ring with a simple socle.

5) For any finite-dimensional \( \mathbb{K} \)-algebra \( R \), the following are equivalent:
   (i) \( R \) is a Frobenius ring
   (ii) There exists a non-singular bilinear pairing \( B: R \times R \to \mathbb{K} \) s.t. \( B(xy, z) = B(x, yz) \) for \( x, y, z \in R \)
   (iii) There exists a linear subspace of codimension 1 \( H \subseteq R \) which contains no nonzero right ideal
   (iv) \( \text{Hom}_R(R, \mathbb{K}) \cong R \) as right \( R \)-modules.
   (v) \( \text{Hom}_R(R, \mathbb{K}) \cong R \) as left \( R \)-modules.

6) \( G \) a finite group, \( \mathbb{K} \) field, \( \mathbb{K}G \) is a Frobenius ring.

7) \( H \) a finite-dimensional Hopf algebra, then \( H \) is F.R.
In coding theory finite rings are used.
Recall G a finite abelian group.
A character $\chi : G \rightarrow \mathbb{C}$ is a function such that
$\chi(g + h) = \chi(g)\chi(h)$

R a finite ring, $\chi$ a character of $R$, +; $\alpha \in R$
$\chi^{(a)}: R, + \rightarrow \mathbb{C}$ defined by $\chi^{(a)}(x) = \chi(\alpha x)$

The set $\hat{R}$ of characters of $R$, + becomes an $(R, R)$-bimodule

Via: $\alpha \in R$ $x \in \hat{R}$ $\alpha . x = \chi^{(a)}$ and $x . \alpha = \chi^{(a)}$

**Theorem (Hirano)**

$R$ a finite ring. Then the following are equivalent:

1. $R \cong \hat{R}$ as left $R$-modules
2. $R \cong \hat{R}$ as right $R$-modules
3. $R$ is a Frobenius ring.

(Honold)

4. $(R^J)^{\hat{R}} \cong \text{soc}(R^\hat{R})$ as left $R$-modules
5. $(R^J)^{\hat{R}} \cong \text{soc}(R^\hat{R})$ as right $R$-modules

**Examples**

- Finite fields $\mathbb{F}_q$, $\chi(x) = e^{2\pi i x/q}$
- $\mathbb{Z}/n\mathbb{Z}$, $\chi(x) = e^{2\pi i x/n}$
- Galois rings (Galois extensions of $\mathbb{Z}/n\mathbb{Z}$)
- Finite chain rings
- Products of Frobenius rings
- Matrix rings over $F.R.$
2) MacWilliams identity and MacWilliams extension theorem

**Theorem (Wood)**

Let $R$ be a finite Frobenius ring, $E \subseteq R^m$ a left linear code, then the MacWilliams identity holds:

$$W_E(X, Y) = \frac{1}{|\pi(E)|} \sum_{e \in E} W(R)(X + (1-k)e_1 Y, X-Y)$$

where $\pi(E) = \{ e = (e_1, \ldots, e_m) \in R^m : \sum_{i=1}^m e_i = 0 \mod (1-k) \}$

MacWilliams extension theorem over finite rings

**Theorem (Wood, 1989)**

Let $R$ be a finite Frobenius ring, and suppose $E_1, E_2 \subseteq R^m$ are left linear codes. If $f : E_1 \to E_2$ is an $R$-linear isomorphism that preserves Hamming weight then $f$ extends to a monomial transformation of $R^m$, i.e. there exist $c \in \mathbb{G}_n$ and $a_1, \ldots, a_m \in U(R)$ such that $f(x_1, \ldots, x_m) = (x_1(x_1, a_1, u_1), \ldots, x_m(x_m, a_m, u_m))$ for $x_1, \ldots, x_m \in R$.

In fact finite Frobenius rings are characterized by the extension theorem:

**Theorem (Wood, 2002)** Suppose $R$ is a finite ring. If the extension theorem for Hamming weight holds for linear codes over $R$, then $R$ is a Frobenius ring.
Bibliography

This is a very partial (commented) bibliography
(Uwaga: Przepraszam obiecałem ze to zrobić...dobrze ze nie powiedziałem kiedy).


(2) D. Boucher, F. Ulmer: Linear codes using skew polynomials with automorphisms and derivations, available on Hal since May 31st. Comments: D. Boucher and F. Ulmer

Comments: The codes are defined on a finite field and use inner derivations. Of course, the Ore extension is then of automorphism form but it seems that nevertheless it is interesting to consider this case since the transformation required for erasing the derivation (a change of variables=cv transformations )does not preserve Hamming distance.

(3) Dougherty, self dual codes over commutative Frobenius rings.

Comments: Steven’s web page is also worth to visit:
https://sites.google.com/site/professorstevendougherty/home


Comments: In this paper a characterization of quasi-Frobenius rings using extensions of modules maps between ideals is given in term of extension of maps via multiplication by some elements. It seems to me that this is close enough to the MacWilliams extension theorem to be mentioned here. Actually I am a bit surprised that this paper is nearly never mentioned anywhere.

(6) Jay A. Wood Applications of finite Frobenius Rings to the foundations of coding theory This is one of the many papers Jay Wood has written on the subject.

Comments: It seems to me that it is better (and easier) to redirect you to his web page which contains many interesting papers and PDF of slides from talks he gave around the world. So here is the URL: http://homepages.wmich.edu/~jwood/