When mutually epimorphic modules are isomorphic

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Abstract

The well known Schröder-Bernstein Theorem states that any two sets with one to one maps into each other are isomorphic. The question of whether any two (subisomorphic or) direct summand subisomorphic algebraic structures are isomorphic, has long been of interest. Kaplansky asked whether direct summands subisomorphic abelian groups are always isomorphic? The question generated a great deal of interest. The study of this question for the general class of modules has been somewhat limited. Dehghani, Azmy and Rizvi in 2019 [4], extended the study of this question for modules.
In addition, a dual of the Schröder-Bernstein Theorem is that two sets with surjection maps onto each other are isomorphic. Analogous to this dual, the question of whether two algebraic structures which are epimorphic to each other, are always isomorphic to each other, is of interest. For modules over a given ring, this does not hold true in general. For a ring R, a subclass $\mathcal{C}$ of R-modules is said *to satisfy the dual of Schröder-Bernstein (or DSB)* property if any pair of its members are isomorphic whenever each one is epimorphic image of the other. In this paper, we investigate the DSB property for the classes of (quasi-)discrete and (quasi-)projective modules among other results. We also investigate the DSB property for the class of injective modules. As applications, our investigations provide answers to several open questions raised in [9].

Keywords

Discrete module, DSB property, epimorphic image, projective module, quasi-discrete module, quasi-projective module, subisomorphic

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