Rings Related to R-projectivity and Max-projectivity

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a joint work with

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Abstract

Let R be a ring with an identity. A right R-module is said to be R-projective if every homomorphism from M to R/I can be lifted to a homomorphism from M to R, for each right ideal I of R. M is said to be max-projective if the said property holds for each maximal right ideal of R. As it is well known R is QF if and only if each injective right R-module is projective. This characterization of QF rings and the given two generalization of projectivity leads to the following definitions that are studied in [1]. A ring R is said to be almost-QF (respectively, max-QF) if each injective right R-module is R-projective (respectively, max-projective). Several classes of rings that are almost-QF and max-QF are given in [1]. In this talk, I will mention generalization of some results and address some questions that are given in [1]. We obtain characterizations of max-QF rings for local rings, semilocal right semihereditary and for right nonsingular right Noetherian rings. We show that being almost-QF and max-QF are not left-right symmetric.

Keywords

R-projectivity, max-projectivity, QF-rings, almost-QF rings, max-QF rings.

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