Hopf algebras versus Hopf heaps

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Abstract

In [1] we defined *a Hopf heap* as an algebraic system $(C, \Delta, \varepsilon, [-,-,-])$ consisting of a coalgebra (C, Δ, ε) with comultiplication $\Delta\colon C \to C \otimes C$, $x \mapsto \sum x_1 \otimes x_2$, and a coalgebra map $[-,-,-]\colon C \otimes C^{co} \otimes C \to C$, $x \otimes y \otimes z \mapsto [x, y, z]$ satisfying associativity $[[x, y, z], t, u] = [x, y, [z, t, u]]$ and generalized Mal'cev identities $\sum [x_1, x_2, y] = \sum [y, x_1, x_2] = \varepsilon(x)y$ for all $x, y, z, t, u \in C$. Starting with any Hopf heap $(C, \Delta, \varepsilon, [-,-,-])$ and any group-like element $e \in G(C)$ we can assign a Hopf algebra $(C, \Delta, \varepsilon, \cdot_e, e, S_e)$ to it by setting $\cdot_e\colon C \otimes C \to C$, $x \cdot_e y = [x, e, y]$ and $S_e\colon C \to C$, $S_e(x) = [e, x, e]$ for all $x, y \in C$. Conversely, every Hopf algebra $(H, \Delta, \varepsilon, \cdot, 1, S)$ gives rise to a Hopf heap $(H, \Delta, \varepsilon, [-,-,-]_\cdot)$ by taking the ternary operation $[-,-,-]_\cdot\colon H \otimes H^{co} \otimes H \to H$, $[x, y, z]_\cdot = x \cdot S(y) \cdot z$ for all $x, y, z \in H$. A Hopf heap can be understood as a Hopf algebra in which the identity element has not been specified. A choice of any group-like element in the Hopf heap can reduce the ternary operation to a binary operation that makes the underlying set into a Hopf algebra in which the chosen group-like element is the identity element. This talk is intended as a discussion of Hopf heaps.

Keywords

Hopf algebras, Hopf heaps

References

[1] T. Brzeziński, M. Hryniewicka, *Translation Hopf algebras and Hopf heaps*, https://doi.org/10.48550/arXiv.2303.13154.