Growth rate of monomial algebras as a derived invariant of noncommutative varieties

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Abstract

A graded associative algebra A can be considered as a coordinate ring of some "noncommutative variety". If the algebra is Noetherian or at least coherent, the construction due to Artin an Zhang [1] associates to it a *noncommutative scheme* of coherent sheaves. The role of these sheaves is then played by the quotient category $\mathsf{qgr}(A)$ of the category of graded finitely presented A-modules by the finite-dimensional ones. Moreover, one can consider its bounded derived category $\mathbf{D}^b(\mathsf{qgr}(A))$. Can one then recover some characteristics of the algebra from invariants of this triangulated category? We show that for monomial algebras, one can recover the exponential growth rate of it, that is, its entropy.

Let A be a finitely presented associative monomial algebra, that is, the quotient of a path algebra by an ideal generated by a finite collection of paths. We calculate the categorical entropy [2] of the Serre twist functor on $\mathbf{D}^b(\mathsf{qgr}(A))$ and show that it is equal to the (natural) logarithm of the entropy of the algebra A itself. Moreover, we relate these two kinds of entropy with the topological entropy of the Ufnarovski graph of A and the entropy of the path algebra of the graph. If A is a path algebra of some quiver, the categorical entropy is equal to the logarithm of the spectral radius of the quiver's adjacency matrix.

Surprisingly, for algebras of polynomial growth the analogous results do not hold. Namely, in the case of monomial algebras of polynomial growth the polynomial categorical entropy [3] is always zero and does not depend on the Gelfand–Kirillov dimension of the algebra.

The talk is mainly based on the paper [4].

Keywords

Coherent ring, monomial algebra, noncommutative scheme, categorical entropy.

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References

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