Structure of admissible pairs

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based on joint work with M. Akian, S. Gaubert, L. Gatto, J. Jun, and K. Mincheva

Abstract

This is part of an ongoing project originally motivated by a search with Itzhakian and Knebusch for a suitable algebraic structure for tropical mathematics which led to supertropical algebras. This leads one to search for the most fundamental structure which still supports a robust theory. In 2016 the author introduced “triples” and “systems,” which in this talk are generalized to the bare essentials: An *admissible pair* $(\mathcal{A}, \mathcal{A}_0)$ over a multiplicative monoid $(\mathcal{T}, \mathbf{1})$ is a $\mathcal{T}$-module $\mathcal{A}$ containing $\mathcal{T}$, additively generated by $\mathcal{T}$, together with a $\mathcal{T}$-subset $\mathcal{A}_0$. The set $\mathcal{A}_0$ plays the role of a zero element, and we assume that there is an element $a \in \mathcal{T}$ (not necessarily unique) such that $\mathbf{1} + a \in \mathcal{A}_0$. In a virtual talk at Kiev, this year, we described joint work with Akian and Gaubert on linear algebra over pairs. In this introduction we describe joint work with Jun, Mincheva, and Gatto on algebraicity, the geometry of pairs, including the analogy of the prime spectrum, and Lie theory. Of special interest are “metatangible pairs,” in which the sum of two elements of $\mathcal{T}$ is in $\mathcal{T} \cup \mathcal{A}_0$. (These include classical algebras, where $\mathcal{A}_0 = \{\mathbf{0}\}$, supertropical algebra, and hyperrings.)

Keywords

Algebraic pairs, geometry, hyperrings, Lie, metatangible, prime spectrum, supertropical, system.

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