

# Nil clean group rings over metacyclic groups

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## Abstract

A ring is nil clean if each of its elements is the sum of an idempotent and a nilpotent. Previously in [J. Cui, Y. Li and H. Wang. On nil clean group rings, *Commun. Algebra* 49(2) (2021) 790-796], the nil clean group rings over dihedral groups and generalized quaternion groups were completely determined. In this paper, we investigate group rings  $RG$  over more general metacyclic groups  $G$ , where  $G = \langle a, b \mid a^n = b^m = 1, b^{-1}ab = a^r \rangle$ . We provide a new criterion for a general nil clean group ring based on the upper central series of  $G$  consisting of 2-groups, and whether the group ring over its respective factor group is nil clean. We also determine values of  $m$  and  $n$  (as a prime power) in order for  $RG$  to be nil clean, and verify whether their corresponding group rings are indeed nil clean.

## Keywords

Nil clean ring; group ring; metacyclic group; Wedderburn-Artin theorem.