On rings whose every nonzero left ideal is an intersection of finitely many maximal left ideals

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Abstract

A ring R is called left PFS if left proper factors of R are semisimple. We prove that R is a left PFS ring if and only if every nonzero left ideal of R is an intersection of finitely many maximal left ideal. We will observe that being PFS is not a left-right symmetric property. Moreover, we observe that a left PFS ring is either a semisimple ring or a ring with only one non-zero left ideal or a domain. We show that R is a left PFS domain if and only if every R-module is a direct sum of an injective and a torsion free modules. When R is a domain, R is a left PFS ring if and only if R is a left SI-ring if and only if R is a left V-ring with K-dim$R$=1 if and only if R is a left PCI ring. In case R has only one left ideal, it is a strictly left weakly V-ring, and also R is a strictly left and right weakly V-ring if and only if R is a left and right PSF ring. Among other things, we prove that a ring R is a Camillo-Krause left weakly V-ring if and only if R is a left PFS ring. This provides a partial answer to Camillo-Krause problem.

Keywords

SI-ring, PCI ring, strictly left weakly V-ring and Camillo-Krause ring

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